The results for sodium–potassium are plotted on Subbotin's curve in Fig. 4. Agreement is fairly good although the present sodium–potassium results generally lie below Subbotin's curve at larger values of y^{++}/y^{++}_{max}. The sodium–potassium results would fit Subbotin's curve slightly better if ψ is defined in terms of the RMS of the turbulent temperature fluctuation rather than the mean square value.

The effect of different fluid Prandtl number on the turbulent temperature fluctuations is shown in normalized form in Fig. 5 with Laufer's [10] velocity data shown for comparison. As the Prandtl number decreases, the molecular conduction effects are enhanced. The region of interplay between the conduction and convection effects therefore dominate the flow field, evident by the broad maximum exhibited in sodium–potassium temperature data, with the opposite true for large Prandtl number fluids such as ethylene glycol. Only in the case of air, with a fluid Prandtl number of 0·704, are the temperature and velocity fluctuation curves similar, as expected from analogy models.

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VIEW FACTOR BETWEEN TWO HEMISPHERES IN CONTACT AND RADIATION HEAT-TRANSFER COEFFICIENT IN PACKED BEDS

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NOMENCLATURE

AS IS well known, the view factor between two bodies is

A, surface area of hemisphere [m^2];
A_C, cross sectional area of sphere [m^2];
F_{ij}, view factor from surface i to j;
$\bar{F}_{12}$, view factor from surface 1 to 2, including the effect of refractory surfaces, see equation (3);
$\mathscr{F}_{12}$, view factor to allow for interchange between surfaces 1 and 2, see equations (4) and (5);
h_r, radiation heat transfer coefficient [kcal/m^{2}h °C];
h'_r, based on A_C, see equation (9);
p, emissivity of hemisphere;
Q, rate of heat transfer by radiation [kcal/h];
T, absolute temperature [°K];

T_1, T_2, of surfaces 1 and 2;
$\bar{T}$, $=(T_1 + T_2)/2$;
X, radius [m].

Greek symbols
$\beta, \gamma, \theta, \phi$, angles;
σ, Stefan–Boltzmann constant, $4{\cdot}88 \times 10^{-8}$ [kcal/m^2 h °K^4].

expressed as

$$A_1F_{12} = \int F_{dA_1-A_2}\,dA_1 = \int_{A_1}\int_{A_2} \frac{\cos\phi_1 \cos\phi_2\, dA_2\, dA_1}{\pi L^2} \quad (1)$$

where dA_1 and dA_2 represent surface elements of the two spheres (radius X_1 and X_2, respectively), L is the distance between the two elements, the ϕ's are the angles between the respective normals to the surface elements and the line L, $F_{dA_1-A_2}$ is the radiation view factor from the surface element dA_1 to the whole area A_2, and F_{12} is that from A_1 to A_2.

For the geometric configuration of the two hemispheres shown in Fig. 1(a), the $F_{dA_1-A_2}$ is evaluated by measuring the projected area PS on the tangential plane MN.

$$F_{dA_1-A_2} = \frac{1}{\pi}\left(\gamma - \frac{1}{2}\sin 2\gamma\right) \pm \frac{ab}{2} + \frac{b}{a\pi}\left(e[\sqrt{(a^2 - e^2)}] + a^2 \sin^{-1}\frac{e}{a}\right) \quad (2)$$

where $\gamma = \sin^{-1}\sqrt{(\sin^2\beta_1 - \cos^2\beta_1 \tan^2\beta_2)}$;

$a = \pm[\cos\beta_1 \cos\beta_2 - \cos(\beta_1 \pm \beta_2)]$;

$b = \sin\beta_1$;

$e = \cos\gamma - \cos\beta_1 \cos\beta_2$;

$m = X_2/X_1$;

$$\beta_1 = \cos^{-1}\sqrt{\left[\frac{2(1+m)(1-\sin\theta)}{(1+m)^2 + 1 - 2(1+m)\sin\theta}\right]};$$

$$\beta_2 = \cos^{-1}\frac{(1+m)\cos\theta}{\sqrt{[(1+m)^2 + 1 - 2(1+m)\sin\theta]}};$$

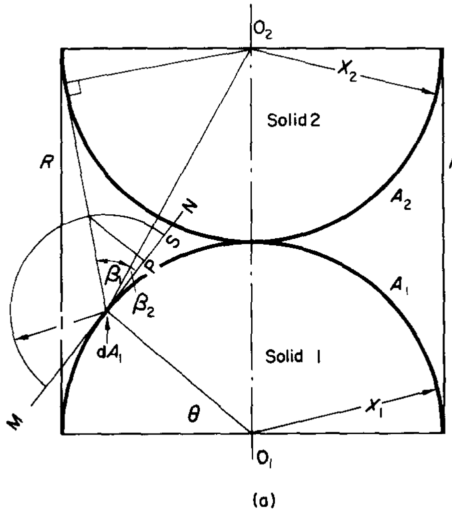
$$\theta_c = \tan^{-1}\frac{1}{\sqrt{[(1+m)^2 - 1]}};$$

($\pm$: minus sign for $\theta = 0 \sim \theta_c$, and plus sign for $\theta = \theta_c \sim \pi/2$). Figure 1(b) shows the computed values of $F_{dA_1-A_2}$, from which F_{12}'s are evaluated as

X_2/X_1	1	1·5	2	3	5	10
F_{12}	0·1511	0·2191	0·2734	0·3520	0·4438	0·5424

Suppose a unit model of radiant heat transfer in beds packed with uniformly sized spherical particles is considered as the two hemispheres (of $m = 1$) circumscribed with a diffusively reflective cylindrical wall, R. The view factor $\bar{F}_{12}$ with reflective wall is

$$\bar{F}_{12} = F_{12} + \frac{F_{1R}F_{R2}}{1 - F_{RR}}. \quad (3)$$



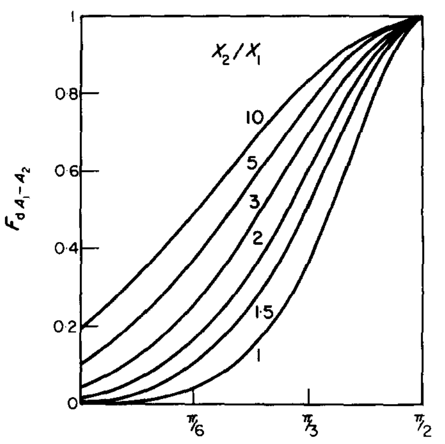


FIG. 1. Coordinates and view factors between two hemispheres in contact.

Considering $F_{1R} + F_{12} = 1$ and $F_{RR} + 2F_{R2} = 1$, one obtains $\bar{F}_{12} = 0{\cdot}576$.

The radiation heat exchange Q between hemisphere 1 (uniform surface temperature T_1) and hemisphere 2 (uniform surface temperature T_2) is expressed as

$$Q = A_1 \mathscr{F}_{12} \sigma(T_1^4 - T_2^4). \tag{4}$$

The overall exchanging factor $\mathscr{F}_{12}$ is [1]

$$\frac{1}{\mathscr{F}_{12}} = 2\left(\frac{1}{p} - 1\right) + \frac{1}{\bar{F}_{12}} \tag{5}$$

where p is the emissivity of the surface of hemispheres.

In terms of radiant heat-transfer coefficient h_r, equation (4) is approximately expressed as

$$Q = A_1 h_r (T_1 - T_2) \tag{6}$$

$$h_r = 4\sigma \mathscr{F}_{12} \bar{T}^3 \tag{7}$$

where $\bar{T} = (T_1 + T_2)/2$ in °K.

Substituting the Stefan–Boltzmann constant $\sigma = 4{\cdot}88 \times 10^{-8}$ (kcal/m^2 h °K^4) and equation (5), equation (7) is rewritten as

$$h_r = \frac{0{\cdot}1952}{2/p - 0{\cdot}264}\left(\frac{\bar{T}}{100}\right)^3 \text{kcal/m}^2\text{h}^\circ\text{C}. \tag{8}$$

Most of the expressions for radiation heat-transfer coefficient have been defined on the basis of cross sectional area A_C.

$$\left.\begin{aligned} Q &= A_C h_r'(T_1 - T_2) \\ h_r' &= 4\sigma\psi\bar{T}^3 \end{aligned}\right\} \tag{9}$$

Thus, the formulae for ψ are compared as follows:

Investigator	Formula for ψ	
Damköhler [2]	$\frac{1}{(2/p) - 1}$	also used in literature [3, 4]
Schotte [5]	p	
Chen and Churchill [6]	$\frac{2}{a + 2b}$	a and b are experimental coefficients
This work	$\frac{2}{(2/p) - 0{\cdot}264}$	2 in numerator is the ratio A_1/A_C

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LAMINAR COUETTE FLOW WITH HEAT TRANSFER NEAR THE THERMODYNAMIC CRITICAL POINT

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NOMENCLATURE

- Br, Brinkmann number, $\frac{\mu_w U_0^2}{k_w(T_0 - T_w)}$;
- C_f, skin fraction coefficient, $2\tau_w/\rho_w U_0^2$;
- h, distance between upper and lower plates;
- k, thermal conductivity;
- Nu, Nusselt number, $\frac{q_w h}{k_w(T_w - T_0)} = -q_w^*$;
- P, pressure;
- q, heat flux;
- Re, Reynolds number, $\frac{\rho_w U_0 h}{\mu_w}$;

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